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$$\therefore s = -\frac{1}{2} \left[\frac{a^\beta (a-x)^{1-\beta}}{1-\beta} + \frac{a^{-\beta} (a-x)^{1+\beta}}{1+\beta} \right] + C_3. \quad \text{For } x=0, s=0.$$

$$\therefore C_3 = \frac{a}{1-\beta^2}. \quad \therefore s = \frac{a}{1-\beta^2} - \frac{1}{2} \left[\frac{a^\beta (a-x)^{1-\beta}}{1-\beta} + \frac{a^{-\beta} (a-x)^{1+\beta}}{1+\beta} \right].$$

Since for $x=a$, $y=b$, we have $b = \frac{a^\beta}{1-\beta^2}$; or, substituting $a=100$, $b=300$, we have $\beta^2 + \frac{1}{3}\beta = 1$; whence $\beta = \frac{1}{6}(\sqrt{37}-1)$ and length of curve between A and $C = \frac{a}{1-\beta^2} = 50(\sqrt{37}+1) = 354.135$.

$$y = \frac{na}{n^2-1} - \frac{na^{1/n}}{2(n-1)} (a-x)^{(n-1)/n} + \frac{n}{2(n+1)} \frac{1}{a^n} (a-x)^{(n-1)/n}.$$

315. Proposed by C. N. SCHMALL, New York City.

If $y=f(x)$, show by Taylor's Theorem that

$$f\left(\frac{x}{1+x}\right) = y - \frac{x^2}{1+x} \frac{dy}{dx} + \frac{x^4}{2(1+x)^2} \frac{d^2y}{dx^2} - \frac{x^6}{2.3(1+x)^3} \frac{d^3y}{dx^3} + \dots \text{ etc.}$$

Solution by the PROPOSER.

$$\text{Put } \frac{x}{1+x} = x+h, \text{ then } f\left(\frac{x}{1+x}\right) = f(x+h),$$

$$\text{and also, } h = \frac{x}{1+x} - x = -\frac{x^2}{1+x}.$$

$$\therefore h^2 = \frac{x^4}{(1+x)^2}, \quad h^3 = -\frac{x^6}{(1+x)^3}, \text{ and so on.}$$

Substituting these values of the powers of h in Taylor's series, we have the required result; i. e., from

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \text{etc.},$$

we get, by direct substitution,

$$f\left(\frac{x}{1+x}\right) = y - \frac{x^2}{1+x} \frac{dy}{dx} + \frac{x^4}{2(1+x)^2} \frac{d^2y}{dx^2} - \frac{x^6}{2.3(1+x)^3} \frac{d^3y}{dx^3} + \text{etc.}$$

316. Proposed by C. N. SCHMALL, New York City.

$$\int_0^\infty \frac{\cos ax}{1+x^2} dx = \frac{1}{2} \pi e^{-a} = \int_0^\infty \frac{x \sin ax}{1+x^2} dx.$$

(From Bromwich, *Theory of Infinite Series*, p. 442, ex. 5, and also from Carslaw, *Fourier's Series*, p. 113, ex. 12.) Prove this by any method.